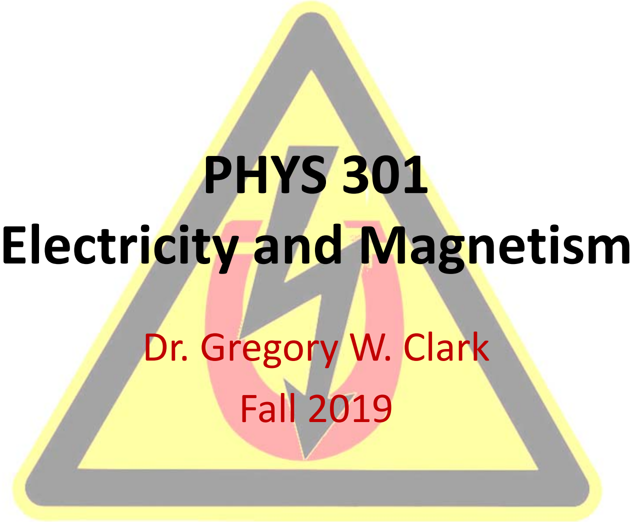


PHYS 301
Electricity and Magnetism

Dr. Gregory W. Clark
Fall 2019

SPS Meeting!

- Thursday, 19 Sept in SCIC 114 at 8:00 pm
- Plan the academic year’s activities
- Elect officers (you could be one!)

SPS SOCIETY OF PHYSICS STUDENTS

- Quiz!!
- Spherical coordinates
- Integral vector calculus

Today!

Maxwell's Equations: Differential Form

- The fundamental theorems of curls and divergences can be used to express Maxwell's equations in differential form:

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_o} \qquad \vec{\nabla}\cdot\vec{B} = 0$$

$$\vec{\nabla}\times\vec{E} = -\frac{d\vec{B}}{dt} \qquad \vec{\nabla}\times\vec{B} = \mu_o\vec{J} + \mu_o\varepsilon_o\frac{d\vec{E}}{dt}$$

using $q \equiv \int \rho \, d\tau$ *and* $I \equiv \int \vec{J}\cdot d\vec{A}$

charge density

current density

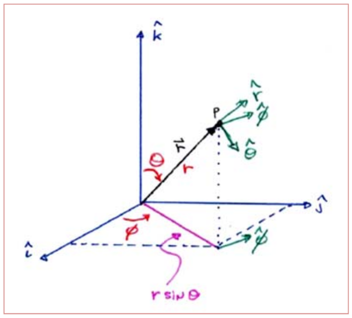
SPHERICAL COORDINATES

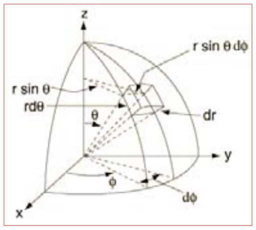
r = radial distance

θ = polar angle

φ = azimuthal angle

UNIT VECTORS : $\hat{r}, \hat{\theta}, \hat{\phi}$





$x = r \sin\theta \cos\phi$

$y = r \sin\theta \sin\phi$

$z = r \cos\theta$

$r = [x^2 + y^2 + z^2]^{1/2}$

$\theta = \tan^{-1}[\sqrt{x^2 + y^2}/z]$

$\phi = \tan^{-1}[y/x]$

$\hat{r} = \sin\theta\cos\phi\,\hat{i} + \sin\theta\sin\phi\,\hat{j} + \cos\theta\,\hat{k}$

$\hat{\theta} = \cos\theta\cos\phi\,\hat{i} + \cos\theta\sin\phi\,\hat{j} - \sin\theta\,\hat{k}$

$\hat{\phi} = -\sin\phi\,\hat{i} + \cos\phi\,\hat{j}$

$\hat{i} = \sin\theta\cos\phi\,\hat{r} + \cos\theta\cos\phi\,\hat{\theta} - \sin\phi\,\hat{\phi}$

$\hat{j} = \sin\theta\sin\phi\,\hat{r} + \cos\theta\sin\phi\,\hat{\theta} + \cos\phi\,\hat{\phi}$

$\hat{k} = \cos\theta\,\hat{r} - \sin\theta\,\hat{\theta}$

$d\vec{l} = dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\,d\phi\,\hat{\phi}$

$d\tau = dV = r^2 \sin\theta\,dr\,d\theta\,d\phi$

$\Big\}$ $d\vec{A}$ DEPENDS ON SURFACE!

DIV, GRAD, and CURL are no longer so simple!!

Integral Calculus

- What is this familiar friend called?

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

where $f(z) = \dfrac{dF(z)}{dz}$

and *f(z)* is continuous on *[a, b]*

The Fundamental Theorem of Calculus

Integral Calculus

- The **fundamental theorem of calculus**:

$$\int_a^b f(x)\,dx = \int_a^b \left(\frac{dF(x)}{dx}\right) dx = F(b) - F(a)$$

- How integrate? Find function *F(x)* whose derivative if *f(x)*!

- NOTE FORMAT:

$\int$ some derivative = value of function at “boundary”

Integral Calculus: Gradients

- The **fundamental theorem of gradients**:

$$\int_{\substack{a\\ \text{line}}}^{b} \left(\vec{\nabla} T\right) \cdot d\vec{l} = \int_{\substack{a\\ \text{line}}}^{b} dT = T(b) - T(a)$$

- Integral is *PATH DEPENDENT*!

- Note that $\oint \left(\vec{\nabla} T\right) \cdot d\vec{l} = 0$